Correspondence

A Deterministic Capacity-Finding Method for Multi-Valued Exponential BAM

Chua-Chin Wang, Chenn-Jung Huang, and Shiou-Ming Hwang

***Abstract*—The multi-valued exponential bidirectional associative memory (MV-eBAM) has been proposed and proved to be asymptotically stable under certain constraints. Although multi-valued eBAM has been verified to possess high capacity by thorough simulations, the capacity is still unable to be solved analytically. In this paper, an algorithm is proposed to derive the capacity. Some important characteristics, including the absolute lower bound of the radix, and the approximate capacity are also discussed. The result shows that the multi-valued eBAM indeed possesses high capacity.**

***Index Terms*—Bidirectional exponential memory, exponential BAM, multi-valued.**

I. Introduction

The bidirectional associative memory (BAM) proposed by Kosko [1], [2] allows an associative search for stored stimulus-response pairs (X_i, Y_i). The storage capacity for perfect recall of the BAM is limited by the number of neurons. Jeng *et al.* [3] proposed one kind of exponential BAM. Although the impressive capacity of an eBAM has been estimated [4], the data representation of BAM or eBAM is still limited to be either bipolar vectors or binary vectors. Based upon the multi-valued concept applied in Hopfield network for analog-to-digital (A/D) conversion [5], Wang *et al.* proposed the multi-valued exponential bidirectional associative memory [6]. In this work, the data range was expanded from $\{-1, +1\}^n$ to $\{1, 2, \cdots, L\}^n$, $L \gg 1$, the asymptotic stability was proved, the high capacity of MV-eBAM was verified using the result of several simulations. However, critical features about this kind of network, such as the lower bound of the radix and the estimation of the capacity, were not thoroughly explored. In this paper, an algorithm to derive the capacity of MV-eBAM will be proposed, the minimum of the radix used to recall all of the stored pattern pairs correctly will be computed, and the approximation of the capacity will be shown analytically and graphically.

II. Framework of MV-eBAM

Suppose we are given M pattern pairs, which are $\{(X_1, Y_1), (X_2, Y_2), \cdots, (X_M, Y_M)\}$, where $X_i = (x_{i1}, x_{i2}, \cdots, x_{\rm in}), Y_i = (y_{i1}, y_{i2}, \cdots, y_{ip})$, where n is assumed to be smaller or equal to p without loss of robustness.

Manuscript received May 10, 1999; revised October 13, 2000. This work was supported in part by the National Science Council under Grant NSC 88-2219-E-110-001. This paper was recommended by Associate Editor S. Lakshmivarahan.

C.-C. Wang and S.-M. Hwang are with the Department of Electrical Engineering, National Sun Yat-Sen University, Kaohsiung, Taiwan 80424, R.O.C.

C.-J. Huang is with the Department of Computer Science and Information Education, National Taitung Teachers College, Taitung, Taiwan 95004, R.O.C.

Publisher Item Identifier S 1083-4427(00)11083-5.

Hence, according to [6], the evolution equations of the MV-eBAM are shown as

$$y_k = H\left(\frac{\sum_{i=1}^{M} y_{ik} b^{-\sum_{j=1}^{n}|x_{ij}-x_j|}}{\sum_{i=1}^{M} b^{-\sum_{j=1}^{n}|x_{ij}-x_j|}}\right)$$
$$x_k = H\left(\frac{\sum_{i=1}^{M} x_{ik} b^{-\sum_{j=1}^{p}|y_{ij}-y_j|}}{\sum_{i=1}^{M} b^{-\sum_{j=1}^{p}|y_{ij}-y_j|}}\right) \quad (1)$$

where

X and Y	input key patterns;
b	positive number, called the radix with $b > 1$;
x_k and x_{ik}	kth digits of X and X_i with y_k and y_{ik} for Y and Y_i, respectively;
$H(\cdot)$	staircase function shown as the following equation

$$H(x) = \begin{cases} 1, & x < 1 \\ L, & x > D \\ \dfrac{L}{D}\cdot x + 0.5, & \text{elsewhere} \end{cases} \quad (2)$$

where

l	$= 1, 2, \cdots, L$;
L	number of finite levels;
D	finite interval of the staircase function.

The reasons for using an exponential scheme in (1) are to enlarge the attraction radius of every stored pattern pair and to augment the desired pattern in the recall reverberation process. In the evolution equations in (1), if the given input pattern is close to the desired pattern, the weighting coefficient, $b^{-\sum_{j=1}^{n}|x_{ij}-x_j|}$, will be close to the maximum, 1, while if the input pattern is far from the desired one, it will approach 0. As for the purpose of the denominator, it makes the y_k and x_k to be the centroids of all of the y_{ik}'s and x_{ik}'s, respectively.

In order to precisely compute the capacity of the MV-eBAM, we have to consider the worst case of pattern distribution. Assume that all of the stored pattern pairs are unique, and the given input pattern is the same as either the X vector or Y vector of one of the pairs. Thus, according to (1), we can take one of the evolution equations as an illustration. Let X_h be an input vector which means Y_h is the vector to be retrieved.

See (3) at the bottom of the next page where the first term in the $H(\cdot)$, i.e., y_{hk}, is deemed as the signal, and the second term is treated as the noise. In order not to make the y_k jump to either of y_{hk}'s adjacent levels, the sufficient condition is

$$-\frac{1}{2} < \frac{\sum_{i\neq h}(y_{ik}-y_{hk})b^{-\sum_{t=1}^{n}|x_{it}-x_{ht}|}}{1+\sum_{i\neq h} b^{-\sum_{t=1}^{n}|x_{it}-x_{ht}|}} < \frac{1}{2}. \quad (4)$$

That is, the noise must be bounded. Therefore, the discussion can be divided into two parts. The right part and the left part of the inequality shown in (4) can be simplified, respectively, to the following inequalities:

$$\sum_{i\neq h}[2(y_{ik}-y_{hk})-1]\cdot b^{-\sum_{t=1}^{n}|x_{it}-x_{ht}|}<1$$
$$\sum_{i\neq h}[2(y_{hk}-y_{ik})-1]\cdot b^{-\sum_{t=1}^{n}|x_{it}-x_{ht}|}<1.$$

In summary, the above two inequalities are rewritten as the following equation:

$$\sum_{i\neq h}[2|y_{hk}-y_{ik}|-1]\cdot b^{-\sum_{t=1}^{n}|x_{it}-x_{ht}|}<1. \tag{5}$$

Let us consider the distribution under the worst condition, $|y_{hk}-y_{ik}|=L-1$. Then (5) is again to be restated as

$$\sum_{i\neq h}(2L-3)\cdot b^{-\sum_{t=1}^{n}|x_{it}-x_{ht}|}<1. \tag{6}$$

The worst case for the pairs distribution happens when those X_i, $i\neq h$, are located as close to X_h as possible. This will produce the largest noise to the signal y_{hk}. For instance, if $n=p=2$, the worst condition to X_h is shown in Fig. 1. Assume r is the largest number of different digits between any X_i, $i\neq h$, and X_h in the worst case of pattern pairs distribution. In addition, the number of level used in the staircase function $H(\cdot)$ also affects the distribution of the pattern pair. Thus, assume $m(n,d,l)$ is the largest number of patterns satisfying $|X_h-X_i|=d$, where d is the number of different digits between X_h and X_i, and l will be defined in the following text. Then the worst case of the pattern pairs distribution must be

$$\sum_{d=1}^{r-1}m(n,d,l)\leq M-1<\sum_{d=1}^{r}m(n,d,l) \tag{7}$$

where $l=L/2$ if L is even, and $l=((L-1)/2)$ if L is odd. Therefore, according to (7), (6) can be rewritten to be the following equation:

$$(2L-3)\cdot\left[\sum_{d=1}^{r-1}m(n,d,l)*b^{-d}+\left(M-1-\sum_{d=1}^{r-1}m(n,d,l)\right)*b^{-r}\right]<1. \tag{8}$$

The definition of the absolute lower bound of the radix can be stated as follows: the smallest radix which is able to recall every uniquely stored pattern pair. In other words, we are interested in discovering what is the minimal radix that is good enough to recall every pattern pair as long as these stored pairs are one-to-one associated. Therefore, we have to consider the worst case in order to derive this minimal radix, which is called **the absolute lower bound** (ALB). The significance of ALB is stemmed from the VLSI implementation of the exponential scheme of MV-eBAM. One of the successful realization of the exponential scheme used in (1) is using the subthreshold current in MOSFET transistors [7]:

$$I_D\propto I_0\,\exp\left[\frac{q(V_G-V_T)}{Mk_BT}\right] \tag{9}$$

where

- I_0 current constant;
- V_T threshold voltage;
- M slop factor.

However, in any real implementation, the dynamic range of the exponentiation circuit will be constrained [8]. Therefore, the performance of this model is the case of fixed dynamic range needs to be analyzed. Suppose the dynamic range of the exponentiators is fixed at D, where

$$D\propto b^n. \tag{10}$$

Then as n grows, b decreases. This means if we can derive the minimum radix, we can have a maximum dimension for the stored vectors. Hence, the exploration of ALB of the radix is very critical when it comes to the physical VLSI implementation for such a neural network.

In fact, the computation of ALB is tightly correlated to the capacity analysis. We also have to consider the worst-case pattern distribution. Hence, (8) gives an inequality to compute the minimal b for the MV-eBAM.

III. Computation of Capacity and the Absolute Lower Bound

Since (8) is hard to be solved analytically, the numerical method is necessary. A deterministic algorithm to derive the capacity, M, is sketched as follows:

```
Given n, b, L;
Let l = ⌊L/2⌋, M = 1, V = (1/2L − 3), d = 1;
While (m(n, d, l) * b^{-d} < V)
  V = V − m(n, d, l) * b^{-d};
  M = M + m(n, d, l);
  d = d + 1;
End while
M = M + ⌊V * b^d⌋
```

$$y_k=H\left(\frac{\sum_{i=1}^{M}y_{ik}b^{-\sum_{t=1}^{n}|x_{it}-x_{ht}|}}{\sum_{i=1}^{M}b^{-\sum_{t=1}^{n}|x_{it}-x_{ht}|}}\right)=H\left(\frac{y_{hk}+y_{hk}\sum_{i\neq h}b^{-\sum_{t=1}^{n}|x_{it}-x_{ht}|}+\sum_{i\neq h}(y_{ik}-y_{hk})b^{-\sum_{t=1}^{n}|x_{it}-x_{ht}|}}{1+\sum_{i\neq h}b^{-\sum_{t=1}^{n}|x_{it}-x_{ht}|}}\right)=H\left(y_{hk}+\frac{\sum_{i\neq h}(y_{ik}-y_{hk})b^{-\sum_{t=1}^{n}|x_{it}-x_{ht}|}}{1+\sum_{i\neq h}b^{-\sum_{t=1}^{n}|x_{it}-x_{ht}|}}\right) \tag{3}$$

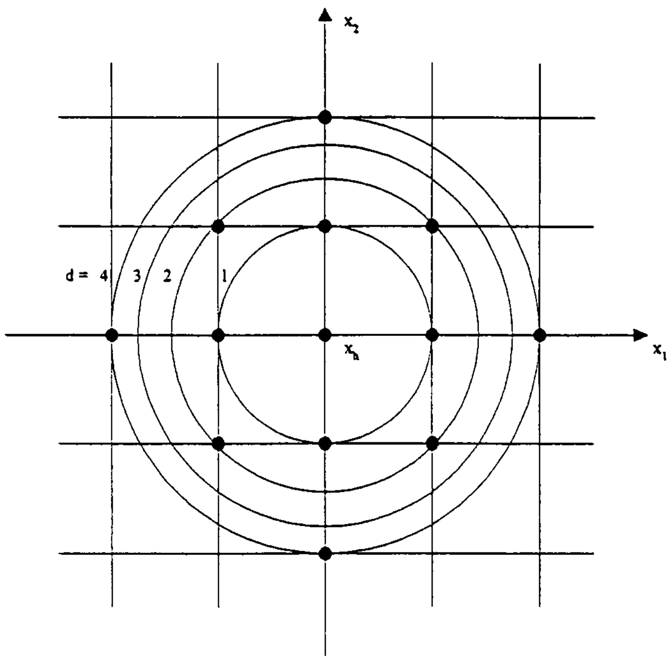

Fig. 1. Worst distribution of MV-eBAM $(n = p = 2)$.

A family of curves for M, b, L when $n = 8$ can be computed as shown in Fig. 2. The family of these curves is similar to the I–V curves of MOS transistors. Therefore, several terms are borrowed to describe the properties of these curves. We call the left side of the curves the "linear region," and right side "saturation region." The result of (8) is dominated by the first term, $m(n, d, l) * b^{-1}$, since $b \gg M$. The boundary of these two regions is located at $M - 1 = m(n, 1) = 2n$, where $b = 2n(2L - 3)$.

A. Linear Region

$M - 1 < 2n$. Eqn. (8) can be simplified to be

$$b \approx (M - 1) * (2L - 3).$$

Thus, the capacity M is rewritten as $M \approx (b/(2L - 3)) + 1$.

B. Saturation Region

$M - 1 > 2n$. Because $b \gg M$ and $b^d \gg m(n, d, l)$, the curves will reach a saturation value when r is getting larger. The relationship between capacity M and the radix b is the same as that of the boundary point, which is

$$b \approx 2n(2L - 3)$$

This implies that when b is larger than the boundary point, it will then approach the maximum number of combinations of the input vector, i.e. $M \approx L^n$. Therefore, the MV-eBAM indeed possesses a high capacity.

IV. Conclusion

In this paper, we have demonstrated the high capacity of MV-eBAM. Since the analytic form of the capacity of the MV-eBAM is hard to derive, a novel method considering the worst case pattern distribution is proposed to estimate the capacity of these multi-valued neural networks. The derivation of the lower bound of the radix provides with us the information about how large the radix should be such that every unique pair can be recalled in the worst case of pattern distribution. In addition to the stability of the MV-eBAM, these characteristics have revealed the potential of this structure to be utilized in a variety of applications. Due to the high capacity and the multi-level feature of this network, the MV-eBAM is potentially useful in the application of data compression, where a reduction of the input pattern space dimension is essential. Moreover, this network is particularly good at learning perceptive type of tasks such as the recognition of complex patterns. Other applications include content-addressable memory, storage of words and of continuous speech, and A/D conversion.

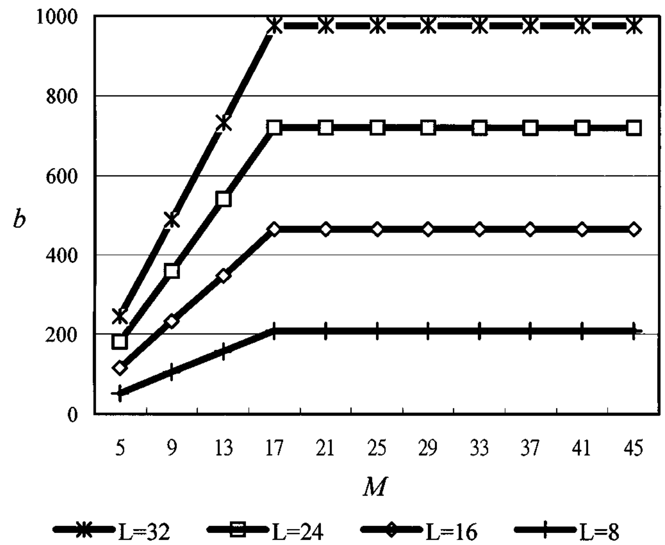

Fig. 2. Lower bound b versus capacity M.

References

[1] B. Kosko, "Adaptive bidirectional associative memory," *Appl. Opt.*, vol. 26, pp. 4947–4960, Dec. 1987.

[2] ——, "Bidirectional associative memory," *IEEE Trans. Syst., Man, Cybern.*, vol. 18, pp. 49–60, Jan./Feb. 1988.

[3] Y.-J. Jeng, C.-C. Yeh, and T. D. Chiueh, "Exponential bidirectional associative memory," *Electron. Lett.*, vol. 26, pp. 717–718, May 1990.

[4] C.-C. Wang and H.-S. Don, "An analysis of high-capacity discrete exponential BAM," *IEEE Trans. Neural Networks*, vol. 6, pp. 492–496, Mar. 1995.

[5] J. D. Yuh and R. W. Newcomb, "A multi-level neural network for A/D conversion," *IEEE Trans. Neural Networks*, vol. 4, pp. 470–483, May 1993.

[6] C.-C. Wang, S.-M. Hwang, and J.-P. Lee, "Capacity analysis of the asymptotically stable multi-valued exponential bidirectional associative memory," *IEEE Trans. Syst., Man, Cybern.*, vol. 26, pp. 733–743, Oct. 1996.

[7] J. Franca and Y. Tsividis, *Design of Analog–Digital VLSI Circuits for Telecommunications and Signal Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1994.

[8] L. A. Glasser and D. W. Dobberpuhl, *The Design and Analysis of VLSI Circuits*. Reading, MA: Addison-Wesley, 1985.