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Short Paper

Pattern Recognition by High-Capacity Polynomial Bidirectional Hetero-Associative Network *

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This investigation presents a novel method of image processing using the polynomial bidirectional hetero-associative network (**PBHAN**). This network can be used for industrial application of optical character recognition. According to the results of detailed simulations, the PBHAN has a higher capacity for pattern pair storage than do the conventional bidirectional associative memories and fuzzy memories. The practical capacity of a PBHAN considering fault tolerance is discussed. The fault tolerance requirement leads to the discovery of the attraction radius of the basin for each stored pattern pair. PBHAN takes advantage of fuzzy characteristics in evolution equations such that the signal-noise-ratio is significantly increased. In this work, we apply the result of this research to pattern recognition problems. The practical capacity of fuzzy data recognition using PBHAN and considering fault tolerance in the worst case is also estimated. Simulation results are presented to verify the derived theory.

Keywords: associative networks, optical character recognition, fuzzy data, neural networks, PBHAN

1. INTRODUCTION

The methodologies used in optical character recognition schemes usually include linear classification, statistical approaches, fuzzy set theory, and many others. Associative memories have been the focus of extensive research in the study of neural networks and pattern recognition [1-3]. In related works, Kosko presented a fuzzy associative memory (FAM) system structure [4]. However, no energy function introduced in his works could ensure that every stored pattern pair would reside at a local minimum on energy surfaces. In this work, we present a novel method of image processing using the polynomial bidirectional hetero-associative network (PBHAN). PBHAN has a higher pattern pair storage capacity and better performance in image processing than do the conventional BAMs or fuzzy memories. Since an image character might be noisy, adding fault tolerance capability to PBHAN facilitates noise immunity. The practical capacity of image processing using PBHAN and considering fault tolerance is estimated here. The derived theory has been applied successfully to OCR.

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2. FRAMEWORK OF HIGH CAPACITY PBHAN

2.1 Evolution Equations

Associated characters or images are digitized or quantized into pattern pairs, which are $\{(X_1, Y_1), (X_2, Y_2), \ldots, (X_M, Y_M)\}$, where $X_i = (x_{i1}, x_{i2}, \ldots, x_{in})$, $Y_i = (y_{i1}, y_{i2}, \ldots, y_{ip})$. Let $1 \le i \le M$, $x_{ij} \in [0, 1]$, $1 \le j \le n$, $y_{ij} \in [0, 1]$, $1 \le j \le p$, n and p be the component dimensions of X and Y, and let $x_{ik}, y_{ik} \in \{0/\lambda, 1/\lambda, \ldots, \lambda/\lambda\}$, fuzzy space = [1, 0], and λ be a fuzzy quantum. The following evolution equations are the recall process of the PBHAN:

$$y_k = H\left(\frac{\sum_{i=1}^{M} y_{ik} \cdot \left((u - \|X_i - X\|^2)/u\right)^{M^Z}}{\sum_{i=1}^{M} \left((u - \|X_i - X\|^2)/u\right)^{M^Z}}\right), \tag{1}$$

$$x_k = H\left(\frac{\sum_{i=1}^{M} x_{ik} \cdot \left((u - \|Y_i - Y\|^2)/u\right)^{M^Z}}{\sum_{i=1}^{M} \left((u - \|Y_i - Y\|^2)/u\right)^{M^Z}}\right), \tag{2}$$

where M denotes the number of patterns in the PBHAN, X_i, Y_i and $i = 1, \ldots, M$ represent the stored patterns, X or Y is the initial vector presented to the network, x_k and x_{ik} denote the kth digits of X and X_i, respectively, y_k and y_{ik} represent the kth digits of Y and Y_i, respectively, Z is a positive integer, u denotes a function defined in the equation

$$u = \sum_{i=1}^{M}\sum_{j=1}^{M}\left(\|X_i - X_j\|^2 + \|Y_i - Y_j\|^2\right), \tag{3}$$

where $u \le C_2^M \cdot (n+p) \le C_2^M \cdot (2n) \le M \cdot (M-1) \cdot n,$ if n is assumed to be larger than p and $H(\cdot)$ is a staircase function shown in the equation

$$H(x) = \begin{cases} 0, & x < 1/(2\lambda) \\ \lfloor x + 1/(2\lambda) \rfloor, & \text{elsewhere} \end{cases} \tag{4}$$

Note that if $\lambda \to \infty$, then $H(x) \approx x,$ for $x \in [0, 1]$. Furthermore, u is bounded according to Eq. (3).

2.2 Energy Function and Stability

The fact that every stored pattern pair should produce a local minimum on the energy surface to be recalled correctly accounts for why the energy function is intuitively defined as

$$E(X, Y) = \sum_{i=1}^{M} \|X - X_i\|^2 \cdot \|Y - Y_i\|^2. \tag{5}$$

The fuzzy data model using PBHAN can be viewed as one kind of BAM, i.e., bidirectional associative memory. Therefore, its stability can be elucidated by closely examining its two phases of evolution, i.e., $X \rightarrow Y$ and $Y \rightarrow X$. In addition, we adopt the SNR approach to compute the theoretical capacity of PBHAN in the worst case [5]. The theoretical minimal capacity of PBHAN without considering fault tolerance in the worst case is

$$M \geq \left\{ \frac{ln(1/2)}{ln((u - (1/\lambda^2))/u)} \right\}^{Z^{-1}},$$

where $u \leq C_2^M n + C_2^M p = C_2^M \cdot (n+p) \leq C_2^M \cdot (2n) \leq M \cdot (M-1) \cdot n$

2.3 Analysis of the Capacity of PBHAN with Fault Tolerance

Considering the required fault tolerance capability, we need to enlarge the area where the stored patterns reside. We introduce a basin concept into the storage of patterns. The radius of the basin where the target pattern is located is called the attraction radius, r. That is, where the distance between X (input pattern) and X_h (the target pattern) is less than or equal to r, we can still recall the target pattern, X_h, and its corresponding pattern, Y_h.

Theorem 1: If $\|X_h - X\| \leq r$ and the PBHAN can recall X_h given X, then the maximal capacity of stored pattern pairs for a fuzzy PBHAN in the worst case is

$$M < M_{\max} = \left\{ \frac{ln((\lambda-2)/\lambda)}{ln((u-r^2)/u)} \right\}^{Z^{-1}}.$$

***Proof*:** According to Eq. (1), we will only discuss the Y part of the evolution equations without any loss of robustness here. The SNR approach is adopted herein to compute the practical capacity of PBHAN. Note that if $\lambda \rightarrow \infty$, then $H(x) \approx x$, for $x \in [0, 1]$. Thus, we can rewrite Eq.(1) as

$$y_k = H\left(\frac{\sum_{i=1}^{M} y_{ik} \cdot \left((u - \|X_i - X\|^2)/u\right)^{M^Z}}{\sum_{i=1}^{M} \left((u - \|X_i - X\|^2)/u\right)^{M^Z}} \right) \approx \left(\frac{\sum_{i=1}^{M} y_{ik} \cdot \left((u - \|X_i - X\|^2)/u\right)^{M^Z}}{\sum_{i=1}^{M} \left((u - \|X_i - X\|^2)/u\right)^{M^Z}} \right),$$

$$y_k \cdot \sum_{i=1}^{M} \left(\frac{u - \|X_i - X\|^2}{u} \right)^{M^Z} = \sum_{i=1}^{M} y_{ik} \left(\frac{u - \|X_i - X\|^2}{u} \right)^{M^Z}. \tag{6}$$

According to Eq. (6), the following equations can be obtained.

$$y_k \cdot \sum_{i=1}^{M} \left(\frac{u - \|X_i - X\|^2}{u} \right)^{M^Z} = \sum_{i=1}^{M} y_{ik} \left(\frac{u - \|X_i - X\|^2}{u} \right)^{M^Z},$$

$$= \; y_{1k} \cdot \left(\frac{u - \left\| X_1 - X \right\|^2}{u} \right)^{M^z} + \; y_{2k} \cdot \left(\frac{u - \left\| X_2 - X \right\|^2}{u} \right)^{M^z} \; + \cdots + \; y_{Mk} \cdot \left(\frac{u - \left\| X_M - X \right\|^2}{u} \right)^{M^z} .$$

The largest noise possible appears in the worst case, in which any X_i, $i \neq h$, is just one component different from X_h (assuming that X is the input pattern and Y_h is about to be recalled). Meanwhile, the other components of X_i and X_h remain the same. For instance, $X_h = (x_{h1}, x_{h2}, \ldots, x_{hn})$, and $X_i = (x_{i1}, x_{i2}, \ldots, x_{in} \pm 1/(2\lambda))$, where x_{ik}, $y_{ik} \in \{0/\lambda, 1/\lambda, \ldots, \lambda/\lambda\}$.

Note that if the distance between X and X_h (the target pattern) is less than or equal to r, then this input pattern, X, can still recall the target pattern, X_h, and its corresponding pattern, Y_h. Considering the required fault tolerance capability, we substitute the $\left\| X - X_i \right\|$ with the attraction radius (r) in Eq. (6). Eq. (6), thus, can be rewritten as

$$\begin{aligned}
&\sum_{i=1}^{M} y_{ik} \left(\frac{u - \left\| X_i - X \right\|^2}{u} \right)^{M^z} \\
&= \sum_{i=h} y_{ik} \cdot \left(\frac{u - \left\| X_i - X \right\|^2}{u} \right)^{M^z} + \sum_{i \neq h}^{M} y_{ik} \cdot \left(\frac{u - \left\| X_i - X \right\|^2}{u} \right)^{M^z} \\
&= y_{hk} \cdot \left(\frac{u - r^2}{u} \right)^{M^z} + \sum_{i \neq h}^{M} y_{ik} \cdot \left(\frac{u - \left\| X_i - X \right\|^2}{u} \right)^{M^z}
\end{aligned}$$

Here, Y_h is assumed to be the desired pattern, and y_k, y_{ik} and y_{hk} represent the kth digits of Y,Y_i and Y_h, respectively. The first term in the above equation corresponds to the signal, and the other terms are the noise. The power of the signal is

$$S = \left[\left(\frac{u - r^2}{u} \right)^{M^z} \right]^2 = \left(1 - \frac{r^2}{u} \right)^{2(M)^z} .$$

Let X_i and Y_i be the stored pattern pairs, and, for the sake of clarity, let

$$P = \left(\frac{u - r^2}{u} \right)^{M^z} \cdot y_{hk} + \sum_{i \neq h}^{M} y_{ik} \cdot \left(\frac{u - \left\| X_i - X \right\|^2}{u} \right)^{M^z} , \tag{7}$$

$$\begin{aligned}
Q &= \left(\frac{u - 0}{u} \right)^{M^z} \cdot y_{hk} + \sum_{i \neq h}^{M} y_{ik} \cdot \left(\frac{u - \left\| X_i - X \right\|^2}{u} \right)^{M^z} \\
&= y_{hk} + \sum_{i \neq h}^{M} y_{ik} \cdot \left(\frac{u - \left\| X_i - X \right\|^2}{u} \right)^{M^z} .
\end{aligned} \tag{8}$$

According to Eq. (6) to Eq. (8), the following inequalities can be obtained:

$$P \leq y_k \cdot \sum_{i=1}^{M} \left(\frac{u - \left\| X_i - X \right\|^2}{u} \right)^{M^z} = \sum_{i=1}^{M} y_{ik} \cdot \left(\frac{u - \left\| X_i - X \right\|^2}{u} \right)^{M^z} \leq Q$$

$$
\begin{aligned}
P &= \left(\frac{u-r^2}{u}\right)^{M^z} \cdot y_{hk} + \sum_{i\neq h}^{M} y_{ik} \cdot \left(\frac{u-\left\|X_i - X\right\|^2}{u}\right)^{M^z} \\
&\geq \left(\frac{u-r^2}{u}\right)^{M^z} \cdot y_{hk} + y_{gk} \cdot \left(\frac{u-\left\|X_g - X\right\|^2}{u}\right)^{M^z}, \quad g \neq h,\ g \in \{1, 2, 3, \ldots, M\}.
\end{aligned} \tag{9}
$$

Let $y_{hk} = i/\lambda$, $y_{gk} = j/\lambda$, i and $j \in \{0, 1, 2, \ldots, \lambda\}$, and y_{hk} and $y_{gk} \in \{0/\lambda, 1/\lambda, 2/\lambda, \ldots, \lambda/\lambda\}$. The expectation values of i and j can also be derived as follows:

$$E(j) = E(i) = \frac{1}{\lambda+1} \sum_{i=1}^{\lambda} i = \frac{1}{\lambda+1} \cdot \frac{1}{2}[\lambda \cdot (\lambda+1)] = \frac{\lambda}{2}.$$

The sufficient condition for the noise to be bounded is

$$y_{hk} - \frac{1}{2\lambda} \leq \left(\frac{u-r^2}{u}\right)^{M^z} \cdot y_{hk} + y_{gk} \cdot \left(\frac{u-1/\lambda^2}{u}\right)^{M^z} \leq P \ .$$

Since we are deriving the inequality of the sufficient condition, the left-hand side of the above inequality indicates where the lower bound is,

$$
\begin{aligned}
&-\frac{1}{2\lambda} \leq \left[\left(\frac{u-r^2}{u}\right)^{M^z} - 1\right] \cdot y_{hk} + y_{gk} \cdot \left(\frac{u-1/\lambda^2}{u}\right)^{M^z} \\
&-\frac{1}{2\lambda} \leq \left[\left(\frac{u-r^2}{u}\right)^{M^z} - 1\right] \frac{i}{\lambda} + \frac{j}{\lambda} \left(\frac{u-1/\lambda^2}{u}\right)^{M^z} \\
&-\frac{1}{2\lambda} \leq \left[\left(\frac{u-r^2}{u}\right)^{M^z} - 1\right] \frac{\lambda/2}{\lambda} + \frac{\lambda/2}{\lambda} \left(\frac{u-1/\lambda^2}{u}\right)^{M^z} \\
&\qquad = \frac{1}{2}\left[\left(\frac{u-r^2}{u}\right)^{M^z} + \left(\frac{u-1/\lambda^2}{u}\right)^{M^z} - 1\right] \\
&\frac{\lambda-1}{2\lambda} \leq \frac{1}{2}\left[\left(\frac{u-r^2}{u}\right)^{M^z} + \left(\frac{u-1/\lambda^2}{u}\right)^{M^z}\right]
\end{aligned} \tag{10}
$$

$$1 - \frac{1}{\lambda} \leq \left(\frac{u-r^2}{u}\right)^{M^z} + \left(\frac{u-1/\lambda^2}{u}\right)^{M^z} \ . \tag{11}$$

In addition, the following inequalities can be obtained from the Eq. (8):

$$Q = \left(\frac{u-0}{u}\right)^{M^z} \cdot y_{hk} + \sum_{i\neq h}^{M} y_{ik} \cdot \left(\frac{u-\left\|X_i - X\right\|^2}{u}\right)^{M^z}$$

$$= y_{hk} + \sum_{i \neq h}^{M} y_{ik} \cdot \left(\frac{u - \|X_i - X\|^2}{u} \right)^{M^z}$$

$$= y_{hk} + y_{1k} \cdot \left(\frac{u - \|X_i - X\|^2}{u} \right)^{M^z}$$

$$+ y_{2k} \cdot \left(\frac{u - \|X_i - X\|^2}{u} \right)^{M^z}$$

$$+ \cdots + \ y_{Mk} \cdot \left(\frac{u - \|X_i - X\|^2}{u} \right)^{M^z}$$

$$\leq y_{hk} + (M-1) \cdot y_{gk} \cdot \left(\frac{u - \|X_g - X\|^2}{u} \right)^{M^z} \ , g \neq h,\ g \in \{1,2,3,\ldots,M\}.$$

The sufficient condition for the noise to be bounded is

$$y_{hk} + \frac{1}{2\lambda} \geq y_{hk} + (M-1) \cdot y_{gk} \cdot \left(\frac{u - 1/\lambda^2}{u} \right)^{M^z} \geq Q \ .$$

Again, since we are deriving the inequality of the sufficient condition, the left hand side of the above inequality reveals where the upper bound is:

$$y_{hk} + (M-1) \cdot y_{gk} \cdot \left(\frac{u - 1/\lambda^2}{u} \right)^{M^z} \leq y_{hk} + \frac{1}{2\lambda} \ ,$$

$$(M-1) \cdot \frac{j}{\lambda} \cdot \left(\frac{u - 1/\lambda^2}{u} \right)^{M^z} \leq \frac{1}{2\lambda} \ , \tag{12}$$

$$(M-1) \cdot \frac{\lambda/2}{\lambda} \cdot \left(\frac{u - 1/\lambda^2}{u} \right)^{M^z} \leq \frac{1}{2\lambda} \ ,$$

$$(M-1) \cdot \left(\frac{u - 1/\lambda^2}{u} \right)^{M^z} \leq \frac{1}{\lambda} \ . \tag{13}$$

Herein, we also take the expectation value of j ($E(j) = \lambda/2$) into the above inequality. Then, we can make the following assumption for the equations easy to read:

$$A = \left(\frac{u - r^2}{u} \right)^{M^z} ,$$

$$B = \left(\frac{u - 1/\lambda^2}{u} \right)^{M^z} .$$

Assume that we take A and B into Eqs. (11) and (13); then, Eqs. (14) and (15) can be derived as follow, respectively:

$$1-\frac{1}{\lambda} \le A+B, \tag{14}$$

$$(M-1)B \le \frac{1}{\lambda},$$

$$B \le \frac{1}{(M-1)\lambda}. \tag{15}$$

According to Eqs. (14) and (15) , the following inequalities can be simplified as follows: Hence, the maximal Z in the worst case for the PBHAN is derived in the following to accurately recall every stored pattern pair is derived as follows:

$$1-\frac{1}{\lambda} \le A+B \le A+\frac{1}{(M-1)\lambda},$$

$$1-\frac{1}{\lambda} \le A+\frac{1}{(M-1)\lambda} \le A+\frac{1}{\lambda} \quad \text{if } M \ge 2,$$

$$\text{A} \ge 1-\frac{2}{\lambda}=\frac{\lambda-2}{\lambda}, \quad \left(\frac{u-r^2}{u}\right)^{M^Z} \ge \frac{\lambda-2}{\lambda}, \tag{16}$$

$$M^Z ln\left(\frac{u-r^2}{u}\right) \ge ln\left(\frac{\lambda-2}{\lambda}\right). \tag{17}$$

Since $ln((u-r^2)/u)$ in Eq. (17) is smaller than zero, we obtain

$$M^Z \le \frac{ln((\lambda-2)/\lambda)}{ln((u-r^2)/u)} \tag{18}$$

$$ln\, M^Z \le ln\left\{\frac{ln((\lambda-2)/\lambda)}{ln((u-r^2)/u)}\right\}$$

$$Z \le \frac{1}{ln\, M}\cdot ln\left\{\frac{ln((\lambda-2)/\lambda)}{ln((u-r^2)/u)}\right\}, \tag{19}$$

where

$$u \le C_2^M n + C_2^M p = C_2^M \cdot (n+p) \le C_2^M \cdot (2n) \le M \cdot (M-1)\cdot n \tag{20}$$

and $n = \min(n, p)$. We deem the above equation to be the absolute upper bound of u. The above Eq. (19) is the upper bound solution of Z, and according to Eq. (18), the practical capacity can be derived as follows:

$$M \le \left\{\frac{ln((\lambda-2)/\lambda)}{ln((u-r^2)/u)}\right\}^{Z^{-1}}. \tag{21}$$

where

$$u = M \cdot (M - 1) \cdot n$$

3. SIMULATION ANALYSIS

The BAM-like associative memory is a two-layer heteroassociator that stores a prescribed set of bipolar library pairs. It consists of two layers of neurons. One layer has n neurons and the other has p neurons. n is assumed to be less than or equal to p without any loss of robustness. The following definitions of n and p from Eq. (22) to Eq. (25) are the same.

Amari and Maginu [6] conducted capacity analysis of the first-order autocorrelator, which has capacity

$$M_{1st-auto} = \frac{n}{2\log n + \log\log n}, \quad n = \min(n, p). \tag{22}$$

Baldi and Venkatesh [7] performed the same analysis for a higher-order autocorrelator, which has capacity

$$M_{HOauto} = \frac{n^{d-1}}{2(d!)\log n}, \quad d = 2. \tag{23}$$

As for the capacity of Kosko's BAM, Haines and Hecht-Nielsen [8] estimated it to be

$$M_{Kosko-BAM} = \frac{r}{2\log r}, \quad r = \min(n, p). \tag{24}$$

Tai et al. [9] also proposed a high-order BAM. They did not estimate or prove the possible capacity of their high-order BAM except, but claimed better recall probability. However, we still can reasonably expect that the capacity of the high-order BAM will be about that of the high-order autocorrelator, because 1) the BAM is intrinsically a variety of the high-order autocorrelator, and 2) if n is large, the capacity shown in Eq. (22) is about the same as that in Eq. (24).

Haines and Hecht-Nielsen [10] proposed another variety of BAM, i.e., the *non-homogeneous* BAM. They enlarged the capacity to be

$$M_{non-ho} = (0.68)\frac{n^2}{(\log_2 n + 4)^2}. \tag{25}$$

Shi et al. [10] proposed a general model for BAMs (GBAM) with associative patterns between the X-space and Y-space. The storage capacity of each of the above BAMs is no larger than n. In contrast, the storage capacity of GBAM is $0.9n$, $1.05n$, $1.1n$ and $1.15n$ for n = 10, 20, 30 and 40, respectively. The capacity of GBAM exceeds n when n is greater than 10 and grows more than linearly as n increases.

Kosko's [4] fuzzy associative memory (FAM) was first to use neural networks to articulate fuzzy rules for fuzzy systems. Despite its simplicity and modularity, this model suffers from extremely low memory capacity, i.e., one single rule per FAM matrix. Furthermore, it is limited to small rule-base applications.

Chung and Lee [11] proposed a multiple-rule storage property for a FAM matrix. They showed that more than one rule could be encoded by the Kosko's FAM. However, they did not derive the maximum capacity of a FAM. The actual capacity depends on the dimension of the matrix and the rule characteristics, e.g., how many the rules overlap. The capacity of this model is limited since it depends on whether or not the membership function is semi-overlapping.

Table 1 presents the capacity (M) of Amari's first-order autocorrelator, Baldi's higher-order autocorrelator, Kosko's BAM (estimated by Hecht-Nielsen), the *nonhomogeneous* BAM, GBAM and PBHAN with λ = 2, respectively. We used PBHAN with λ = 2 to obtain a fair comparison between it and other BAM-like designs which usually process either binary vectors or bipolar vectors. According to the data shown in this table, fuzzy data recognition using PBHAN has high pattern storage capacity.

Table 1. A comparison of capacity (*M*) among Amari's first-order autocorrelator, Baldi's higherorder autocorrelator, Kosko's BAM (estimated by Hecht- Nielsen), the *nonhomogeneous* BAM, GBAM and PBHAN.

n / **Name**	10	20	30	40	50	60	70	80	90	100
AMARI	1.83	2.82	3.73	4.60	5.44	6.25	7.03	7.81	8.56	9.31
Baldi	1.08	1.66	2.20	2.71	3.19	3.66	4.11	4.56	5.00	5.42
Kosko	2.17	3.33	4.41	5.42	6.39	7.32	8.23	9.12	10.00	10.85
non-ho	0.60	3.92	7.71	12.52	18.27	24.94	32.47	40.84	50.03	60.02
GBAM	9	20	33	46	60	75	91	108	126	145
PBHAN ***(λ = 2)***	27	55	83	110	138	166	194	221	249	277

r	*M*	*n*	*r*	*M*	*n*
0.000001	10947	50	0.000001	11756	150
0.000010	5558	50	0.000010	6438	150
0.000100	1591	50	0.000100	2145	150
0.001000	858	50	0.001000	904	150
0.010000	246	50	0.010000	262	150
0.100000	63	50	0.100000	73	150

Fig. 1 reveals that the practical capacity of a PBHAN with the fault tolerance ability drastically decreases with increasing the attraction radius.

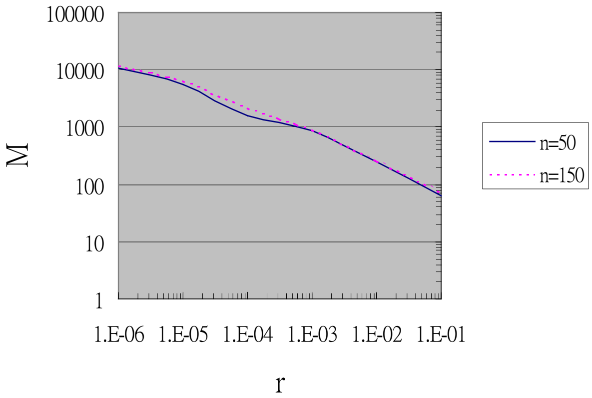

Fig. 1. The practical capacity of PBHAN in the worst case with fault tolerance radius ($\lambda = 3$, $Z = 3$).

Example 1. We applied the results obtained in this research to real character recognition problems. The PBHAN was used to store and recall a set of 7×11 fuzzy data composed of twenty-six different pattern pairs (English letters, upper case and lower case). Fig. 2 presents some pattern pairs with $n = p = 77$ to this network. According to our simulation results, only one iteration was needed for every capital letter to recall its lower case letter correctly, and vice versa.

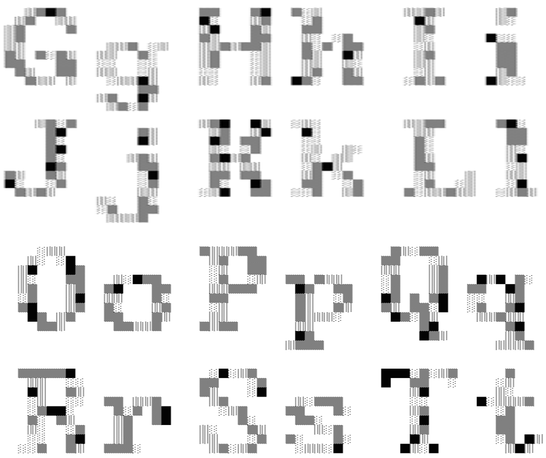

Fig. 2. Pattern recognition examples ($M = 26$, $n = p = 77$).

4. CONCLUSIONS

According to our results, fuzzy data recognition using PBHAN provides enjoys high pattern storage capacity. This method utilizes a fuzzy scheme to increase capacity. The proposed energy function ensures that every stored pattern pair is located in a local minimum of the energy surface. The practical capacity of a PBHAN which considers fault tolerance in the worst case can be estimated, thereby allowing us to predetermine the size of the PBHAN based on the possible capacity requirement.

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